

# Classical memory assisted correlation spectroscopy of quantum signals

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Quantum spectroscopy with single qubits has improved considerably our ability to detect weak signals arising from classical and quantum sources. Recently, the use of additional memory qubits in quantum spectroscopy has allowed a resolution beyond the sensor coherence time to be reached. Alternatively, combining a classical clock allows for spectroscopy of classical fields with a resolution given by the clock coherence time. We present a novel correlation spectroscopy technique with resolution that is limited by the coherence time of the classical clock. This method is capable of screening the classical signal while still being able to detect the quantum one.

**Introduction** — Quantum metrology and quantum sensing [1, 2] are extremely promising research directions which use quantum mechanics to reach the ultimate limits of measurements accuracy. One of the major goals of this field is the measurement of magnetic fields. State-of-the-art magnetometry often relies on dynamical decoupling where fast pulses or continuous fields drive a quantum mechanical system [3–9]. The role of these fields is to decouple the system from the environment, and thus to enhance the  $T_2$  time [10], while at the same time retaining the ability to sense a signal that is on resonance with the pulse rate.

Characterizing the time dependence of signals, either quantum or classical [11–13] has become one of the central goals of quantum sensing in the last few years. The interest in this regime stems from the ability to use it to sense frequencies with increased resolution. In particular correlation spectroscopy has been extensively used in recent years in the field of NV centers in diamond [14–21]. One of these schemes which is termed Qdyne [19] utilizes classical rather than quantum memory. In this Letter we build upon this work and utilize the ability to manipulate classical data in order to construct a protocol with a better sensitivity that targets quantum signals.

We address the problem where a quantum probe interacts with a nearby quantum system with the goal to estimate the energy gaps of the target system. When energy gaps arise from local coupling internal to the quantum system, such as chemical shifts for example, their estimation provides detailed knowledge of the systems physical structure. We address two issues: 1) Armed with the knowledge that only one nearby quantum system has a frequency in a given range, we seek to estimate this frequency. We refer to this task as precision. 2) Knowing that there are a few nearby quantum systems we aim to resolve their energy gaps. We term this task, resolution. In this Letter we present a scheme which distinguishes between classical and quantum signals. Moreover, the precision is improved over state of the art methods by a factor of  $\sqrt{T_{LO}/T_2}$  and the resolution is improved by a factor of  $T_{LO}/T_2$ , where  $T_{LO}$  is the coherence time of the classical clock and  $T_2$  is the coherence time of the sensor.

**The intuition behind the scheme** — Although the protocol is general, here we illustrate it with the example of magnetometry with nitrogen-vacancy (NV) centers in diamond. Optical readout of the electronic spin of the NV center allows for precision sensing of nanoscale magnetic fields. Here we in-

vestigate the frequency precision of an NV center coupled to nearby nuclear spins in diamond (fig. 1 (a)). The measurement protocol is composed of two magnetic field measurements at the beginning and at the end with the NV spin, separated by time  $\tau$  (fig. 1 (b)). In order to understand the idea we assume that we have the ability to couple at will the NV and the  $^{13}\text{C}$ . In this case the NV performs a Ramsey measurement on the  $^{13}\text{C}$ , as in [22]. Suppose the interaction between the systems is  $gS_zI_x$ , where  $\mathbf{S}$  and  $\mathbf{I}$  are the spins of the NV and  $^{13}\text{C}$ . By measuring the NV we project the  $^{13}\text{C}$  on the  $x$  basis. We wait time  $\tau$  and measure the  $^{13}\text{C}$  on the  $x$  basis again and thus we can estimate the frequency of rotation of the  $^{13}\text{C}$ , i.e., its energy gap. As the quantum sensor has no role during the time  $\tau$  the measurement is limited only by ability to keep track of time with a local oscillator. The time-gap  $\tau$  can be extended up to the coherence time of the local oscillator,  $T_{LO}$ . This intuition, however, is only valid in the regime in which  $gT_2$  is of the order of  $\pi/2$ . We address the question of what happens in the weak coupling limit,  $gT_2 \ll 1$ .

In the weak coupling regime the first measurement creates a small coherence (polarization in the  $x$  direction) in the  $^{13}\text{C}$ . After time  $\tau$  this coherence will cause a rotation which we will be able to estimate with a second measurement. In the following we explain the protocol. The Hamiltonian of the system is

$$H = DS_z^2 - \frac{1}{2}\gamma_e B_z S_z - \frac{1}{2} \sum_i \gamma_{n,i} B_z I_{z,i} + \frac{1}{4} \sum_{ik} A_{zk} S_z I_{k,i}, \quad (1)$$

where  $D$  is the NV zero field splitting,  $\gamma_e, \gamma_{n,i}$  are the gyromagnetic ratios of the NV and the  $i$ -th nuclear spin respectively, and  $A_{zk}$  is the hyperfine coupling between the NV and the nuclei where  $i$  runs over the different nuclei and  $k$  over the different directions. We have made the secular approximation and omitted all the terms that couple to the  $S_{x,y}$ .

**Analyzing precision: the problem of one nucleus** — In order to probe the  $I_x, I_y$  operators we drive the system at resonance in an XY8 sequence with a repetition rate,  $1/\tau_p$  close to the Larmor frequency of the nucleus,  $\omega_l = \gamma_{n,1} B_z$  (fig. 1). With the drive the total Hamiltonian reads,

$$H = \omega_0 S_z + \Omega(t) S_X \cos(\omega_0 t) + \omega_l I_z + \tilde{g} S_z I_x. \quad (2)$$

As we use an XY8 sequence  $\Omega(t)$  consists of a set of sharp  $\pi$  pulses, approximated as delta functions, with a spacing of  $\tau_p = \frac{\pi}{\omega_e}$  from one another, where  $\omega_e$  is our estimation of  $\omega_l$ .

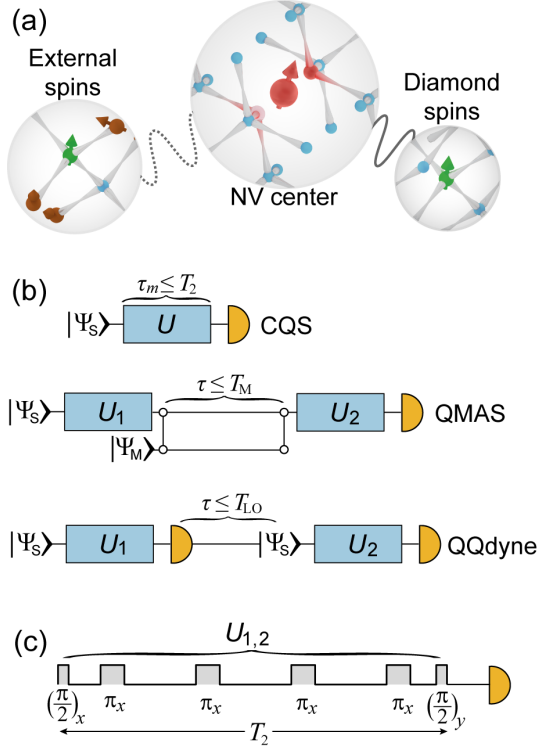


Figure 1. (a) *The central problem.* A quantum sensor which is based on an NV center aims to resolve two quantum systems with very close Larmor frequencies in the presence of classical noise, either inside the bulk diamond or an external spin. (b) Conventional quantum sensing (CQS) exploits dynamical decoupling without the use of correlations and thus the resolution is limited by the  $T_2$  of the NV. Quantum memory assisted sensing (QMAS) methods are limited by the coherence time of the quantum memory. The method we propose (QQdyne) is limited by the coherence time of the classical memory, i.e., the clock. (c) All methods use a dynamical decoupling sequence which reduces the effect of noise and probes a frequency which is close to the frequency of the pulse sequence.

Moving to the interaction picture with respect to  $\omega_0 S_z$ , and assuming  $\Omega(t) \ll \omega_0$ , in rotating wave approximation,

$$H_I = \Omega(t) S_X + \omega_l I_z + \tilde{g} S_z I_x. \quad (3)$$

Thus the Hamiltonian in the interaction picture with respect to the pulses reads:

$$H_{II} = \omega_l I_z + \tilde{g} h(t) S_z I_x, \quad (4)$$

where  $h(t)$  is the square wave:

$$h(t) = \begin{cases} 1 & \omega_e t \bmod 2\pi \in (0, \pi) \\ -1 & \omega_e t \bmod 2\pi \in (\pi, 2\pi). \end{cases}$$

We assume that  $\omega_e$  is very close to  $\omega_l$ , i.e.  $\delta = \omega_l - \omega_e \ll \omega_0$ . In the interaction picture with respect to  $\omega_l I_z$ ,

$$H_{II} = \tilde{g} h(t) S_z (I_x \cos(\omega_l t) + I_y \sin(\omega_l t)). \quad (5)$$

The spin state then evolves during the time  $\tau_m = n \frac{\pi}{\omega_e}$ , which is the spacing between two consecutive measurements and  $n$  is the number of  $\pi$ -pulses applied in this time. Expanding  $h(t)$  into Fourier series and omitting the fast oscillating terms we obtain

$$H_{eff} = g S_z (I_x \cos(\delta t) + I_y \sin(\delta t)), \quad (6)$$

where  $g = (2\tilde{g})/\pi$ .

We aim to measure the NV twice in the  $S_X$  basis, such that the probability is given by:

$$p_{a,b} = \text{Tr}[\Pi_b \mathcal{U}(t_2, t_2 + \tau_m) U_c \Pi_a \rho_i \mathcal{U}_1^\dagger(t_1, t_1 + \tau_m) \Pi_a U_c^\dagger \mathcal{U}^\dagger(t_2, t_2 + \tau_m) \Pi_b] \quad (7)$$

where,  $\rho_i$  is the initial density matrix after the initialization of the NV in the  $|\uparrow_y\rangle$  state and in which the nuclear is in the identity state.  $\Pi_{a,b}$  is the projector onto the  $a, b$  NV states where  $a, b \in \{|\uparrow_x\rangle, |\downarrow_x\rangle\}$ ,  $U_c$  rotates the  $|\uparrow_x\rangle$  state to  $|\uparrow_y\rangle$  (In the experiment  $U_c$  is not applied but the NV is initialized in the  $|\uparrow_y\rangle$  state again) and  $\mathcal{U}(t_i, t_j)$  is the evolution operator defined by the time ordered exponent,

$$\mathcal{U}(t_i, t_f) = T \left\{ \exp \left( -i \int_{t_i}^{t_f} H_{eff}(t) dt \right) \right\}. \quad (8)$$

Specifying to the range  $\delta T_2 \ll 1$ ,  $\tau_m \lesssim T_2$ , we write

$$\mathcal{U}(t_{1(2)}, t_{1(2)} + \tau_m) \approx \exp \left[ -ig T_2 S_z (I_x \cos(\delta t_{1(2)}) + I_y \sin(\delta t_{1(2)})) \right]. \quad (9)$$

Substituting Eq. (9) in Eq. (7) we obtain

$$p_{a,b} = \frac{1}{4} \left[ 1 + s(a,b) \sin^2(\phi/2) \cos(\delta \tau) \right], \quad (10)$$

where  $s(a,b) \in \{1, -1\}$  denotes the sign of the correlation, the dimensionless coupling  $\phi = g T_2$  and  $\tau = t_1 - t_2$ .

As the exact expression for the correlation function Eq. (7) is cumbersome we only present it in a weak coupling limit,

$$p_{\uparrow_x, \uparrow_x} = \frac{1}{4} \left( \delta^{-2} g^2 \sin^2 \left( \frac{\delta \tau_m}{2} \right) \cos(\delta(\tau_m - \tau)) + 1 \right). \quad (11)$$

For  $\delta T_2 \ll 1$  this result agrees with the weak coupling limit of Eq. (10).

Analyzing this probability one obtains a sensitivity of:

$$\Delta \delta = \frac{\sqrt{1 - \sin^4 \left( \frac{\phi}{2} \right) \cos^2(\delta \tau)}}{\sin^2 \left( \frac{\phi}{2} \right) \tau |\sin(\delta \tau)|}. \quad (12)$$

This result can be simply understood: in the relevant regime of weak coupling,  $\phi \ll 1$ , we get for  $\delta \tau = (2n+1) \frac{\pi}{2}$ ,

$$\Delta \delta = \frac{4}{\phi^2 \tau}. \quad (13)$$

In this regime the  $\frac{1}{\tau}$  scaling of a standard Ramsey experiment is obtained, however, as the measurement is weak, there is an

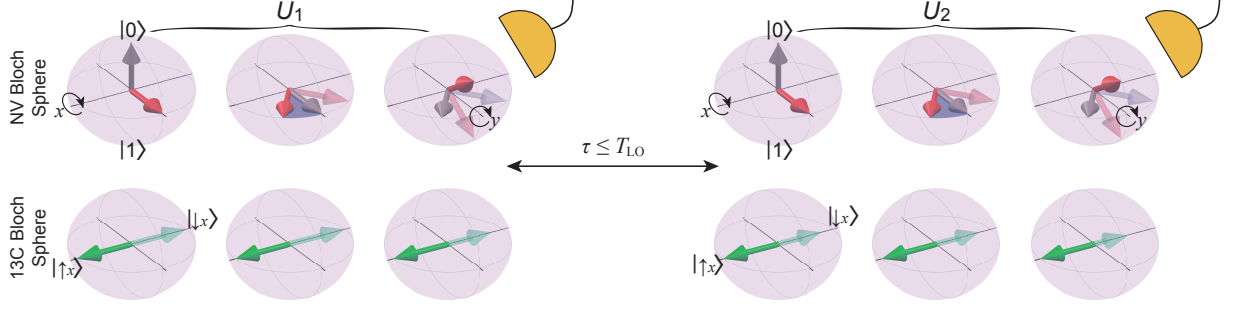


Figure 2. *The explanation of the operation principle of the protocol.* The description of the whole measurements cycle. Upon the initialization the NV spin (red) is polarized along  $|\uparrow_y\rangle$  and the  $^{13}\text{C}$  are in the state with identity density matrix. After time  $\tau_m$  before the first measurement half of the NV population precesses clockwise and another half counterclockwise around the  $z$  axis. Measuring the  $|\uparrow_x\rangle$  state of the NV induces a finite polarization of  $^{13}\text{C}$  along  $\hat{x}$ . After the time  $t_2 - t_1 = \tau = 2\pi\delta^{-1}$  the polarization of  $^{13}\text{C}$  acquired at the first measurements causes more NVs to precess clockwise than counterclockwise. This imbalance is revealed by the second measurement along  $|\uparrow_x\rangle$ . In contrast, if the time interval between the two measurements is  $t_2 - t_1 = (2 + 1/2)\pi\delta^{-1}$  the precession of NVs is determined by nuclei polarization along  $\hat{y}$ , see Eq. (6). Since equal amount of  $^{13}\text{C}$  is polarized along  $|\uparrow_y\rangle$  and  $|\downarrow_y\rangle$  no excess polarization of NV results in this case.

additional small dimensionless prefactor,  $\phi^2$ , which reduces the sensitivity. The standard Ramsey scaling is retrieved for larger  $\phi = (2n + 1)\pi$  we get  $\Delta\delta = 1/\tau$ . This means that for  $gT_2 \geq \pi$ , the  $T_2$  of the probe is not a real limitation. Since  $\tau$  is limited by  $T_{LO}$ , the total sensitivity is given by

$$\Delta\delta = \frac{4}{\phi^2 \sqrt{T_{total} T_{LO}}}. \quad (14)$$

*Intuition behind the procedure* — An intuition for this procedure can be gained by noticing that the NV measurement realizes a quantum non demolition measurement on the nucleus. Two measurements are made in the  $x$  direction with a time separation of  $\tau$ . In the case that the nucleus did not evolve in this time ( $\delta = 0$ ) we will get the same results and hence a high correlation. In the case that the nucleus evolved in between the measurements (large  $\delta$ ) we get different results and diminished correlation.

This reasoning is best explained by considering the NV measurement as a procedure that measures weakly the nucleus along direction in the  $xy$ -plane forming an angle  $\delta t$  with  $\hat{x}$ . Following the above physical picture we observe that a measurement of the NV at time  $t = 0$  projects the NV to the state  $|\uparrow_x\rangle$  and the nucleus to the state,

$$\rho_N(t_1 + \tau_m) = \frac{1}{2} (1 - \sin(\phi/2)) |\downarrow_x\rangle\langle\downarrow_x| + \frac{1}{2} (1 + \sin(\phi/2)) |\uparrow_x\rangle\langle\uparrow_x|, \quad (15)$$

i.e., a slight imbalance between the  $|\uparrow_x\rangle$  and  $|\downarrow_x\rangle$  states, see Fig. 2. The second measurement builds on this imbalance to achieve correlation with the first measurement. The outcome of the second measurement depends on the state of nuclear polarization. For the nucleus in the  $|\uparrow_x\rangle$  state the NV is measured in the  $|\uparrow_x\rangle$  state with probability  $\frac{1}{2} \left(1 + \sin\left(\frac{\phi}{2}\right) \cos(\delta\tau)\right)$ . For the opposite nucleus polarization the NV is measured in the  $|\uparrow_x\rangle$  state with probability  $\frac{1}{2} \left(1 - \sin\left(\frac{\phi}{2}\right) \cos(\delta\tau)\right)$ . This

leads to the correlation function Eq. (10). The contrast is periodic function of  $\delta\tau$ . Indeed, for  $\delta\tau = 2\pi$  the imbalance of nuclear spin polarization along  $\hat{x}$  direction, Eq. (15) induces the excess of NVs in the spin state  $|\uparrow_x\rangle$  compared to the spin state  $|\downarrow_x\rangle$ , see Fig. 2. For  $\delta\tau = 2\pi(1 + 1/4)$ , however the spin precession of NVs during the second measurement is determined by the nuclear polarization along  $\hat{y}$ . As the latter is on average zero, no excess NV polarization along  $\hat{y}$  results, see Fig. 2.

*Resolution* — In order to study resolution we will consider two nuclei. In this case we have a single NV center which interacts with two nuclei having two different detunings from the NV drive  $\delta_1 = \omega_e - \omega_1$ ,  $\delta_2 = \omega_e - \omega_2$ . We define  $\phi_1 = g_1 \tau_m$  and  $\phi_2 = g_2 \tau_m$  and obtain,

$$p_{\uparrow_x, \uparrow_x} = \frac{1}{4} + \frac{1}{16} \cos(\delta_2 \tau) (1 + \cos(\phi_1)) (1 - \cos(\phi_2)) + \frac{1}{16} \cos(\delta_1 \tau) (1 + \cos(\phi_2)) (1 - \cos(\phi_1)). \quad (16)$$

the  $1 + \cos(\phi_i)$  terms are due to the interference of the contributions of the two nuclei. We observe that in the weak coupling limit ( $\phi_1, \phi_2 \ll 1$ ) the signals are independent,

$$p_{\uparrow_x, \uparrow_x} = \frac{1}{4} - \frac{1}{16} \phi_2^2 \cos(\delta_2 \tau) - \frac{1}{16} \phi_1^2 \cos(\delta_1 \tau). \quad (17)$$

We see from this expression that the resolution equals to  $1/\tau$  which is limited by the inverse phase coherence time of the classical clock  $T_{LO}$ . The coherence time of the NV does not limit the resolution but only the sensitivity. The result, Eq. (17) is readily generalized to the case of  $n$  nuclei,

$$p_{\uparrow_x, \uparrow_x} = \frac{1}{4} - \frac{1}{16} \sum_{k=1}^n \phi_k^2 \cos(\delta_k \tau). \quad (18)$$

Correspondingly, our estimates of the resolution apply to the general case.

*Classical vs. quantum signals* — The natural question we address next concerns our ability to differentiate between

quantum and classical signals. As classical signals comprise the major source of noise it is important to differentiate between the two and eliminate the classical noise. Here we show a few methods to differentiate between the two signals.

Here we show that a properly defined correlation function is insensitive to the classical noise and depends solely on quantum correlations. The intuition underlying the construction below builds on the correlations induced by the back-action on the nuclei resulting from the measurement of the NV. These, specifically quantum correlations are not present in the classical case. By analyzing statistically these correlations we reduce the classical noise by a factor of  $\sqrt{N}$ , where  $N$  is the number of measurements performed during one sequence. While the quantum signal is not reduced in this procedure.

Based on this intuition we construct a correlation function as follows. First, we notice that in the classical case the two subsequent measurements are correlated because the phase of the drive is constant during the measurement cycle. These correlations, however are eliminated upon averaging over the noise realizations corresponding to different measurement cycles. To demonstrate this we consider one such measurement cycle. The second measurement takes place only if the first measurement yields a result of  $+1$ , i.e., the NV state is projected on the state  $|\uparrow_x\rangle$ . We show that classical noise suppresses the correlation between the two measurements. Quantum mechanically, however, the second measurement is affected by the first one and we expect finite correlations.

Proceeding classically, we initialize the NV on the state  $\rho_i = |\uparrow_y\rangle\langle\uparrow_y|$ , let the interaction evolve and then post-select on the result  $|\uparrow_x\rangle$ , evolve the system and make the second measurement after time  $\tau$ . The conditional probabilities to obtain post-selectively  $|\uparrow_x\rangle$  and  $|\downarrow_x\rangle$  are defined by Eq. (7).

For a given noise realization,

$$\langle S_x \rangle = \frac{P_{\uparrow_x, \uparrow_x} - P_{\uparrow_x, \downarrow_x}}{P_{\uparrow_x, \uparrow_x} + P_{\uparrow_x, \downarrow_x}} = -\sin(\phi \cos(\theta + \delta\tau)). \quad (19)$$

In order to get this average a series of measurement needs to be performed at the beginning from which the probability for an  $|\uparrow_x\rangle$  result could be deduced for the calculation of  $\langle S_x \rangle$ . This average is finite due to the correlation between the first projection and the second measurement. Crucially, however since the phase  $\theta$  is random the measured result is the average of Eq. (19) over  $\theta$ . This average vanishes,  $\langle \langle S_x \rangle \rangle_\theta = 0$  because the precession angle,  $\phi \cos(\theta + \delta\tau)$  is a random variable with zero mean. To ensure that this quantity averages to zero we perform a few measurements at the beginning and use the average of these measurement to normalize the final mean.

In contrast to the classical case, quantum mechanically, the phase  $\theta$  is fixed by the phase measurement, and therefore is not random. The state of the nucleus is projected weakly in a fixed direction. The initial state prior to the first measurement is  $\rho_i = |\uparrow_y\rangle\langle\uparrow_y| \otimes \mathbb{I}_n$ . With this definition, Eq. (7) yields

$$\langle S_x \rangle = \sin^2\left(\frac{\phi}{2}\right) \cos(\delta\tau). \quad (20)$$

The quantum result, Eq. (20) can be extended to the many-nuclei case similarly to Eq. (18).

It is noteworthy that the post-selection is the crucial part in this protocol as it projects the nuclei to a polarized state. There is, however, no analogue of this effect in the classical scenario.

The above arguments applied in reverse show that it is possible to reduce the quantum noise and to increase quantum signals by performing a repetitive measurements due to the Zeno effect.

A second option is to estimate the phase from the set of measurements at the beginning and then disregard all the estimations which are not zero. As the classical estimation will give the phase of the signal and the quantum will always give zero due to the polarization at the beginning.

As the role of the series of measurement at the beginning is to polarize the nuclei, a natural alternative procedure is to polarize the nuclei at the beginning instead. This can be done by a series of pulses that polarizes the nuclei in the  $|\uparrow_x\rangle$  direction or a Hartmann Hahn sequence which is followed by a  $\pi/2$  pulse on the nuclei. In that case the final measurement result gives the average:

$$\sin\left(\frac{\phi}{2}\right) \sin^2(\phi_p) \cos(\delta\tau), \quad (21)$$

where  $\phi_p$  is the parameter that characterizes the strength of polarization, which in most polarization schemes is of the order of  $\phi$ .

*Increasing the information by repetitive measurements* —

As already mentioned our method is basically a Ramsey experiment with weak measurements, thus the Fisher information of a single experiment is bounded by  $4\tau^2$ , where  $\tau$  is the duration of the experiment. This limit is achieved for large enough  $\phi$ , namely when strong measurements can be applied. However in the weak coupling regime the optimal number of measurements should be examined numerically. An increased number of weak measurements imply stronger initial and final measurements but on the other hand they reduce the accumulation of the phase. We studied this problem numerically, assuming that the nuclear spin is initially polarized. This assumption reduces the complexity, but does not change the problem qualitatively as the role of the first measurements is to polarize the nuclei. The numerical results are shown in fig. (3). It can be seen that there are some competing effects to the increased number of measurements. On the one hand more information is extracted, meaning the weak measurements are converging towards a strong measurement and on the other hand the information is reduced as the time for collecting the phase becomes shorter and due to the finite  $\delta$  the measurements of the nuclei are not in the same direction. Furthermore, due to the oscillatory nature of the probability the Fisher information may oscillate, resulting in more than one peak. Observe that multiple measurements in the middle of the experiment reduce the Fisher information, unlike sensing of classical signals where they give rise to an improved  $\tau^3$  scaling [19]. These weak measurements ruin the accumulation of the phase and it is thus preferable to squeeze them in the end.

As expected, It can be seen from fig. 3 that as the coupling strength is decreased, the optimal number of measurements increases.



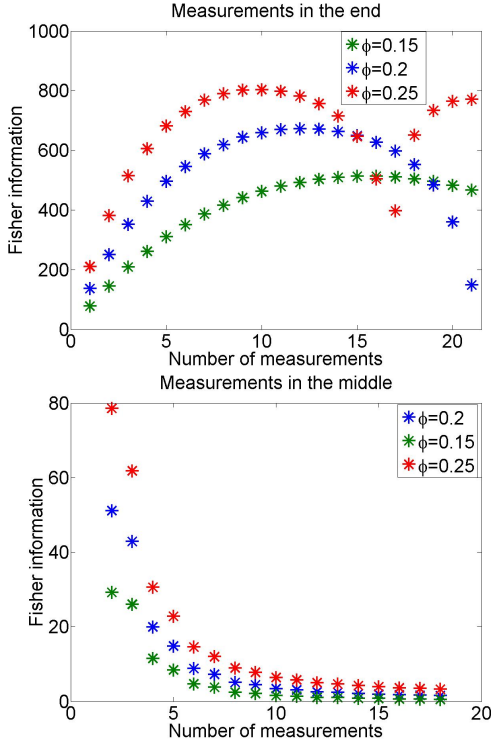


Figure 3. Top: Fisher information (FI) as a function of the number of measurements, where all measurements are squeezed in the end and assuming a polarized state at the beginning. In this plot:  $g = 1$ ,  $\delta = 0.1(g)$ ,  $\tau = 20\left(\frac{1}{g}\right)$ , and  $T_2$  takes the values of 0.25, 0.2, 0.15  $\left(\frac{1}{g}\right)$ . Note that with strong measurement the FI is  $4\tau^2 = 1400\left(\frac{1}{g^2}\right)$ . The reason it is not achieved here is that the time required for a strong measurement is longer than  $\tau$ . Bottom: The FI where measurements are applied in equal spacing during the measurement time with the same parameters as the plot above. In that case multiple measurements just reduce the FI, as they ruin the phase accumulation.

Polarizing the nuclei [23] at the beginning has several advantages over measuring. First, it requires much less scattering of photons which will result in less noise on the nuclei. Second, since no correlation is measured in this method all the classical signal is averaged to zero.

*Estimating the number of nuclei* — A crucial consequence of the difference between the quantum and classical measurements is our ability to estimate the number of nuclei. The

underlying intuition is as follows. The post-selected measurement eliminates the classical signal and thus should decrease with the number of nuclei as in the limit of a large number of nuclei we expect to get the classical result. Moreover, as the sensor is a highly nonlinear system the number of nuclei could be estimated by studying the correlation function, Eq. (19). The correlation function can be calculated analytically for small number of nuclei. As the corresponding expression is cumbersome, here we present the correlation function graphically in Fig. 4. This could be understood by an analogy to an anti-bunching experiment.

Once the number of nuclei is known we are able to estimate

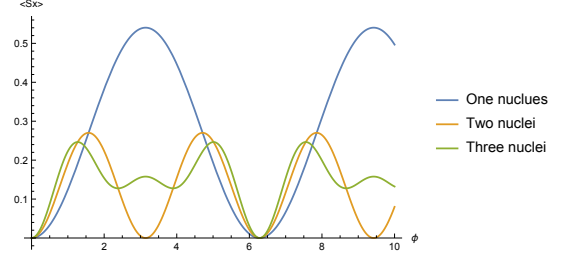


Figure 4.  $S_x$  as a function of  $\phi$  for different number of nuclei. It can be seen that the number of nuclei could be estimated from these results.

the frequencies with any desired accuracy as resolution is no longer an issue. It is important to note that this scheme is not even limited by the coherence time of the nuclei,  $T_2$ , nor its  $T_1$ . This, however requires a large number of measurements as the number of nuclei in the weak coupling limit can only be inferred from higher order terms. Moreover,  $\tau$  should be shorter than  $T_2$ .

*Conclusion and outlook* — We have presented a quantum sensing scheme that enables the detection of quantum systems with a resolution limited by the clock stability. This scheme is able to distinguish between quantum and classical signals and can be applied in a scenario where a single molecule is the subject of sensing and is located close to an NV center, whereas other molecules form a bath of almost classical noise.

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